NONLINEAR STUDIES - www.nonlinearstudies.com
Vol. 21, No. 2, pp. 309-318, 2014

I&S PUBLISHERS

Natural mathematics, computer mathematics, and mathematics: Scope in engineering computation

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Abstract. We discuss the vital characters, including the consistency and its proof aspects, of natural mathematics which knows no error, that of computational/computer mathematics which is, in general, ever entangled with non-separable error, and that of mathematics which attempts to capture/approximate the natural mathematics and involves a combination of an erroneous numerical form and an error-free symbolic form. We then provide the scope and importance of understanding the distinction among these three mathematics with stress on scientific and engineering computations which constitute the interface between the mathematical models and their engineering implementations and which pervade every moment of our daily lives.

1 Introduction

God exists since mathematics is consistent, and the devil exists since we cannot prove the consistency.
Morris Kline (1908-92)

In the same spirit, we write

God exists since computer mathematics is consistent, and the devil does not exist since we can prove the consistency.

and

God exists since natural mathematics is consistent, and the devil does not exist since the inconsistency is completely unknown in nature.

There are three kinds of distinct mathematics, viz., natural mathematics (NMA), computational/computer mathematics (CMA), and regular mathematics (RMA). It is Nature that feeds all the required inputs such as the seed, heat, light, and nutrients, and executes all the functions which outputs a tree. Implicit in these functions is the perfect mathematics used by nature observing all the laws of nature not all exactly known to us or comprehended by us. It is the absolute fact that

2010 **00A05, 97N40, 97N20, 65Y04**

Keywords: Computer mathematics, Engineering computation, Error, Finite precision, Natural mathematics, Regular mathematics, Ultra-high-speed computer

there exists no chaos, no violation of any law, and no fuzziness in nature. The low pressure created over Bay of Bengal and consequent cyclone then entering into coastal cities leaving a trail of destruction of properties and lives have never been whimsical. These have followed all laws and underlying mathematics completely errorlessly, perfectly contradiction-freely and absolutely parallely and have chosen their paths 100 % accurately. We, the human beings, have very inadequate knowledge and computational power to predict their paths sufficiently accurately. A prediction (partial differential equations/Navier-Stokes equations) model designed by humans using their insufficient knowledge and also tools is no match to those of nature. Consequently, the prediction could be completely or partially wrong. Nevertheless we, the mathematicians and computational mathematicians/physicists, are ever learning and ever improving the model and ever attempting to come as close as possible to those activities and NMA. Our mathematics, physics, and computational mathematics may never come reasonably close to NMA/natural science (NSC sciences performed by nature), although we will never cease to pursue and improve in our quest of understanding Nature more and more.

Regular mathematics (RMA) or, simply, mathematics is viewed differently by different people [1-18]. Some of the views are:
(i) Mathematics is the study of quantity. (Aristotle, around 384 - 322 BC)
(ii) Mathematics is the door and key to the sciences. (Roger Bacon, 1214 - 1294)
(iii) Mathematics is the science of order and measure. (Descartes, 1596 – 1650)
(iv) Mathematics is concerned only with the enumeration and comparison of relations. (Gauss, 1777 – 1855)
(v) Mathematics is the science of what is clear by itself. (Jacobi, 1804 – 1851)
(vi) Mathematics today is the instrument by which the subtle and new phenomena of nature that we are discovering can be understood and coordinated into a unified whole. (Homi Jehangir Bhabha (1909 - 66), Indian nuclear physicist).
(vii) Mathematics freeze and parch the mind, . . . an excessive study of mathematics absolutely incapacitates the mind for those intellectual energies which philosophy and life require, . . . mathematics cannot conduce to logical habits at all, . . . in mathematics dullness is thus elevated into talent, and talent degraded into incapacity, . . . and mathematics may distort, but can never rectify, the mind. (Sir William Hamilton (1788-1856), the famed Scottish philosopher, logician, and meta-physicist a negative view that may be construed as a cruel attack on mathematics and hence on mathematicians).

In this context the story "The Blind Men and the Elephant" (from the Buddhist Sutra) is significant. "Why? Because everyone can only see part of the elephant. They are not able to see the whole animal. The same applies to God and to religions. No one will see Him completely." By this story, Lord Buddha teaches that we should respect all other legitimate religions and their beliefs. This famous blind men episode is no disrespect to any mathematician/celebrity. It is only to point out every individual's own limitation to view the endless scope of mathematics in all sciences and engineering in its entirety.

Mathematics is all-pervading and springs from purity of mind. Underlying all scientific/engineering discoveries, we almost always find the validation/justification through mathematics. Whether it is physics or chemistry or biology or civil/ mechanical/ electrical/ electronic engineering or architecture, or computer/space science/technology, it is the mathematics that pervades all these subjects in a completely inseparable manner. If one tries to remove mathematics/mathematical thinking out of these subjects, then these subjects collapse entirely. However, it is the essence of physics and numbers (integer/discrete quantities) that form the pillar of the foregoing science and engineering/ technological subjects. At the base of physics and numbers lies the relevant mathematics/ logical thinking of mathematical nature. It is the endeavour of mathematics and mathematical sciences to come as close as possible to NMA and NSC so that prediction and decision models are continuously improved. Both NMA and RMA have infinity embedded in them. An infinite series such as the series 1+ (1/2) + (1/4) + (1/8) + (1/16) + . . . has the exact finite value 2. The infinite series viz. 1 + (1/2) + (1/3) + (1/4) + (1/5) + . . . , on the other hand, has the value infinity; it is even greater than 10^{100}. One may convince oneself readily using Matlab or possibly even using a pocket calculator. While error is recognized as an important matter in both RMA and CMA for consideration, it is non-existent in NMA. While proofs in mathematics and computational mathematics are essential ingredients for acceptance of statements, such proofs in natural mathematics are not much meaningful to be called for. One and all including mathematicians will accept the outcome of natural mathematics as 100 % perfect, 100 % errorless, 100 % assumption-free, 100 % axiomatic, and 100 % all-natural law abiding. This foregoing statement can never be proved by the conventional mathematical proof procedures since many concerned axioms in nature are either not known to a mathematician or known rather imperfectly. While assumptions whose main purpose is to introduce error and deviation from true happening/outcome so as to make a reasonable solution possible by a human being using the available tools, are essential ingredients in RMA as well as in CMA, these are unknown in NMA. In fact, NMA simply does not need any assumption.

Axioms in nature The nature or, equivalently, the universe is ever dynamic and is completely based on its own mathematics and its perfect unchangeable eternal absolutely correct rules/laws. Newton's three physical laws of motion are axioms (non-dubitable truths for mathematicians) in nature/science. It is not difficult to appreciate the truth by us through our experience/experiment/observation. Thus these laws are assertive sentences which are correct axioms in a macro-level natural activity.

Science is the second system of causes and so is mathematics It may be mentioned that science is the second system of causes and so is mathematics. Consider the question "Why" in the following instances, where these Why's cannot be answered by science (e.g. physics).

(i) Water formation (Two parts of nascent hydrogen reacts chemically with one part of nascent oxygen to make one part of water (H_2O). Why chemical reaction?)
(ii) Gravitational Pull (Why Pull and not Push?)
(iii) Proof by induction/contradiction (Why should induction and/or contradiction prove the theorem?)

Assumption versus axiom The nature of an axiom and that of an assumption are similar. While an axiom is the perceived truth and does not distort the truth, an assumption introduces errors and does distort the truth.

The mathematical proof of any statement is impossible if we are not allowed to take the help of axioms (so called self-evident truths) / assumptions implicitly or explicitly. Thus the proof remains eternal if the concerned assumptions/ axioms remain eternal. "Self-evidence" is not acceptable by a mathematician as a valid proof in mathematics.

We have other different types of proof such as statistical proof, probabilistic proof, documentary proof, evidential proof, and experimental proof. We will not dwell on these proofs as these are not so strictly mathematical proofs nor are these as flawless as mathematical proofs.

Computational Mathematics (CMA) Computational (Computer) mathematics (CMA) is the interface between mathematics (RMA) and real-world engineering/technological implementation and beyond, where mathematics has its own limitations There are three types of CMA Numerical (mostly arithmetic operations on numbers), Semi-numerical (roughly 50 % arithmetic and 50 % non-arithmetic operations), and Non-numerical (operations such as testing, branching, looping). Here we confine ourselves to only numerical aspects of computational mathematics connected with modern digital computers.

CMA differs from RMA in that it does not include infinity. It, however, includes unique absolute (mathematical) zero as well as numerical zero and has a finite range -10^{100} to -10^{-50}, 0, $+10^{-50}$ to $+10^{100}$ (say). A division by the absolute zero in CMA is considered to be a serious violation or, equivalently, a blunder; This is also so in both NMA and RMA, although such division by zero in NMA never occurs. Even division by a numerical zero (a relative zero which is non-unique) is not permitted in CMA (in the context). Since NMA is essentially out of bound, we focus more on RMA and CMA.

In Section 2, we compare RMA with CMA. Ultra-high Speed Computing (UHC) with Dynamic Domain of Applications is discussed in brief in Section 3 while Section 4 mentions emerging areas/problems in engineering and the effect of dynamically changing face of computational mathematics to tackle these problems along with exponentially galloping computational speed, storage space, and band-width. Section 5 comprises conclusions.

2 RMA versus CMA (Numerical)

The exponential growth that the world has witnessed during the last five decades (1960s onward) in architecture, engineering, and technology, specifically, information technology has become possible due to fantastic progress in both computing devices (hardware) and the software. Every eighteen months processor speed is doubling. Every twelve months band-width is doubling and every nine months hard disk space is doubling. Behind all these exponential growth is the computational mathematical algorithms (in the form of software) superimposed on the hardware. Here we can see the real power of RMA through its computational arm. All sciences and all engineering that we see today would have remained dwarf without this computational arm of mathematics If we cut this arm, the whole of todays scientific world would simply collapse.

Inputs and outputs make a vital difference Inputs to RMA and that to CMA along with their outputs make a vital difference between them. CMA deals with **only finite precision** and will continue to do so through eternity; there is always a gap in between any two points. RMA, on the other hand, encompasses both finite as well as infinite precisions while NMA eternally works on **only infinite precision**. One may visualize 2-digit finite rational points (denoted here by asterisks on a (2-D) plane by creating the two-dimensional figure using Matlab commands >> **n=99;t=0:1/n:1,x=t; y=round(rand(n+1, 1)*(n+1))/(n+1), size(y), maxi=max(y), mini=min(y), plot(x, y, '*')**. If the precision of the computer is 2 digits, then the foregoing finite (non-negative) rational numbers produced by the commands could be the inputs in computational mathematics. There are distinct gaps (not necessarily equal; as a matter of fact, often unequal) in between any two points. These gaps should not be construed as random. On the other hand, in NMA, real numbers (totality of rational and irrational numbers) when represented as blue/black points on a white plane depict no gap between any two points. Hence the plane is entirely filled-in and thus is an absolutely non-empty continuous (non-white) blue/black space (plane).

A point and a building block of matter: analogous ? It is interesting to distinguish between a physicist' s concept of a building block of matter and that of a point representing a real number. A particle physicist has been in the quest of finding an indivisible building block of matter where no further division of this block exists. He has never visualized (or even imagined) matter as something having continuous mass not made of tiny particles or, equivalently building blocks with spaces/gaps between any two blocks/particles.

A New Zealand physicist Ernest Rutherford (1871-1937), and a Danish physicist Niels Bohr (1885-1962), developed a way of thinking about the structure of an atom in which an atom looks very much like our solar system. It is known as the Rutherford-Bohr model of Atomic Structure, and was something of a breakthrough in describing the way the atom works. According to this atomic model, an atom consists of positively charged nucleus made of protons and neutrons. Electrons which are equal in number of protons revolve around the nucleus in various orbits. This atom is extremely stable due to electromagnetic force between positively charged nucleus and negatively charged electrons. This model was accepted by scientists, a broad one not involving hundreds of subatomic elementary particles such as mesons, leptons, neutrinos, keons, pions, quark, w plus, w minus, and Higgs Boson (initially theorized in 1964 while its existence tentatively confirmed on March 14, 2013) some are low mass, some others are medium mass, and the rest are heavy mass observed later over decades [19].

However, at a room temperature (usually 24^0C (75.2^0F or $297.15K$), the electrons revolving at different orbits around the nucleus are at relatively very large distances like our planets orbiting the sun at different orbits. When temperature is increasingly brought down, the distance between an electron and the nucleus increasingly becomes short. Also the kinetic energy of a revolving electron decreases and the volume of the atom reduces. When the temperature is brought down to 0 K (rather very close to 0 K as exactly 0 K is not reachable), the whole atom reduces to a mass having numerical zero volume and zero energy state. In the scale of Kelvin, there is no negative temperature unlike that of Celsius and Fahrenheit. It may be remarked in this context that while there is the lower bound of temperature viz. 0 K, there is not yet the known upper bound of temperature beyond which there cannot be any temperature attainable.

A system at absolute zero still possesses quantum mechanical zero-point energy, the energy of its ground state. The kinetic energy of the ground state (or the concerned entropy) cannot be removed. However, in the classical interpretation, it is zero and the thermal energy of matter vanishes. Scientists have achieved temperatures extremely close (within millidegrees) to absolute zero, where matter exhibits quantum effects such as superconductivity and super-fluidity. The matter, when subjected to withdrawal of heat increasingly such that the temperature reaches 0 K, possibly returns to a shapeless and an attributeless phenomenon. In a devolution, it reverts just to potential energy form to manifest as something not exactly experienced by us [20]. Will a physicist be able to appreciate the concept of point representing a real number (infinity of them and uncountable even in an extremely small space/plane) of RMA/NMA in terms of an analogy of a physical phenomenon? However, the concept of a point (finite in number and countable) in CMA can be easily appreciated by a physicist or for that matter, by anybody in science and engineering and possibly even an analogy of this point could be seen by a scientist/engineer in the physical world.

In RMA, a 3-dimensional point is defined as something which has no width, no length, and no height and yet it exists or needs to be assumed to exist for the purpose of all sciences and engineering activities. It is more of a concept that one conceives rather than a definition that appears contradictory. A foregoing building block that a physicist talks about may be somewhat (not completely) analogous to a point. In this respect, a mathematicians view of a real number is not analogous to an elementary particle (since a real number could have infinity of digits). The following table (Table 1) depicts vital differences between RMA/NMA and CMA.

Table 1 Mathematics/Natural Mathematics (RMA/NMA) Versus Computational Mathematics (CMA): Vital Differences

Mathematics (RMA/NMA)	**Computational Mathematics (CMA)**
Domain has ∞ (infinity) of points.	Domain has finite no. f of points.
2-D. domain has ∞^2 of points each represented by two ordered mathematically real numbers.	2-D domain has f^2 of points each represented by two ordered rational numbers, each with finite number of digits.
Proof methods implicitly assume n-D domain having ∞^n points.	Proof methods may simply check validity for each point of n-D domain with f^n points.
Methods of induction/ deduction/ contradiction/ construction form the backbone of mathematical proof. NMA needs no such methods for proof and even no proof; all its statements are absolutely error-free perfect axioms.	These methods are not essential. Exhaustive verification provides the proof. In UHC, such a verification for a specified precision is always possible unless it is not too intractable (e.g. Chess problem).
Assumptions only introduce mathematical error and deviation from the true happenings. NMA is completely assumption-free while RMA is not.	Both assumptions and numerical computations introduce error and deviation more than what RMA projects.
Mathematics in Natural Computer (NMA) is 100 % errorless, assumptionless and only axiom-oriented. RMA encompasses errors, assumptions, and also axioms.	Mathematics in a man-made computer (CMA) is intrinsically erroneous and includes more assumptions than RMA besides axioms.
Natural Computer is the most parallel and fastest computer in the universe started infinite years ago, working now, and will continue to work infinite years hence.	Artificial modern computer has relatively very little parallelism, and too slow. It started about seven decades back (1940s) and will continue for a finite time as long as civilization lives.
Natural Computer knows no break-down and hence no maintenance.	Artificial computer knows occasional break-down and hence preventive and break-down maintenances.
Natural Computer needs no resources/help from humans/ any living being. It operates 100 % independently and is never stoppable.	Man-made computer needs resources and help from humans. It operates depending on availability of resources (e.g. electricity) and humans and is ever stoppable.

How CMA solves RMA problems easily: Example If there is a break (consisting of more than one point or, equivalently, infinity of points) or a jump discontinuity (as per RMA), the CMA finds out and tells us. As a matter of fact, Matlab does this by finding the computational limit of the function at the given point from both sides (from the left as well as from the right) in a 1-D domain if the function does not exist or undefined mathematically (0/0 form or infinity/infinity form or infinity/0 form) at this given point. In a 2-D domain, i.e., for the function f(x, y), we compute using Matlab, four values $f(x \text{ plus or minus } \Delta x, y \text{ plus or minus } \Delta y)$ to find out the limit. Similarly, we can compute the limit of multi-dimensional functions.

If an engineer/a non-mathematician, who wants mathematics not as a strict discipline but as a tool for real-world usage, is asked the value of the function $f(x) = (x^2 - 4)/(x-2)$at $x = 2$, his answer will be 4. But a mathematician's answer will be "undefined". It may be remarked that a mathematical solution such as a solution of the Laplace (partial differential) equation $u_{xx} + u_{yy} = 0$ viz. $u(x,y) = e^x siny$ is of no use to an engineer unless this is converted to numbers subject, of course, to the appropriate numerical boundary conditions. The domain of problems that can be solved using CMA is much larger than that using RMA. In fact, if a problem is not solvable computationally, it cannot be solved by any other means for the use by an engineer.

Godels incompleteness theorem; Blow to complete non-fuzziness of mathematics David Hilbert

(1862-1943), a great German mathematician, proposed at the beginning of twentieth century, 23 problems which, he believed, needed to be solved in all parts (of Hilberts program) to put solid logical foundation under all of mathematics. Kurt Godel (1906-78), a brilliant Austrian logician, showed in a proof that any part of mathematics at least as complex as arithmetic can never be complete [16]. No algorithm, howsoever large, can lead to sorting out all the true or untrue statements/information/equations within a system. He demonstrated that statements exist that cannot be derived by the rules of arithmetic proof.

A simple explanation of Godel's Theorem A statement P which states that there is no proof of P. If P is true then there is no proof of it. If P is false then there is a proof that P is true which is a contradiction. Therefore, it cannot be determined within the system whether P is true. For further details, refer [12, 16,].

3 Ultra-high Speed Computing with Dynamic Domain of Applications

The fastest computer chip, announced by IBM, in 2011, that integrates both electrical and optical nano-devices on the same piece of silicon could soon make it possible for ultra-high speed computers to execute 10^{18} (i.e., one million trillion) flops (floating-point operations per second). This chip is based on IBM's CMOS Integrated Silicon Nanophotonics (CISN) technology at Semicon, Japan.

The information processing of the brain is extremely fast, highly parallel, and much less mistake-prone in certain situations such as the one, where one is to recognize a given picture of her mother which she does within a fraction of a second. A Cray super-computer, on the other hand, would take several seconds to come out with an answer that the picture is that of her mother. In other situations, however, the information processing by the brain of a common human being could be slow and could involve error/mistake. In a conscious state of mind, the processing is often sequential (non-parallel), slow, and could involve error while in a subconscious/unconscious state of mind, the processing could be parallel, fast, and less error-prone. There exists no living being who/which could claim that she/it could process information all the time error-free. As a matter of fact To err (mistake) is human (living being) while not to err is computer (non-living being). could be an extension of the age-old proverb To err is human.

So far as nature/material universe is concerned, it is not difficult to appreciate that perfect mathematical activities involving exact real numbers is continuously going on with the highest possible parallelism and in the fastest possible manner. These mathematical activities of the universal/natural computer is never stoppable and is completely error-free, totally maintenance-free, absolutely assumption-free, as well as entirely bug-free. The concerned natural mathematics was existing infinite years ago, it is existing now, and it will continue to exist infinite years hence and follows all the laws of nature all the time perfectly.

The digital computer is based on silicon technology. That is the only technology which alone has revolutionized the whole world since mid-twentieth century. Both the technological and architectural innovations in silicon technology over decades are responsible for dynamic and exponential increase of processing power, band-width, and storage space (executable memory and hard-disk space). The power is further enhanced significantly by the use of several central processing units as well as other processors/channels/input-output controllers. Moreover, unlike the main-frame days when many users would use one large computer in a time-sharing mode, we have numerous (hundreds of millions) personal computer, both desk-tops and laptops, with a huge processing power available to the world. Only a small fraction of this power is utilized. The remaining huge processing power unutilized is a huge waste. The life of a computer is not reduced by using this untapped power, nor the cost of processing is increased by using all of the power a situation very much unlike our daily consumption of electricity, water, gas, and other necessities. Consequently, many computing problems such as NP-hard problems that were intractable have now become tractable within a reasonable precision and a reasonable size with considerable practical importance. Evolutionary as well as trial and error approaches including genetic algorithms are being more increasingly employed for highly compute-intensive problems with relative error computation to ascertain the quality of the solution. Even in many situations such as the computation of an integral involving complex trigonometric and special functions, such an approach produce better quality of the integration value than that produced by a good quadrature formula [13, 17]. The exhaustive search over a reasonably large finite practical domain is tractable. Consequently the validity of any statement (lemma/ theorem/ corollary) can be checked in CMA; no mathematical proof methods such as those of induction, contradiction, deduction, construction and a combination of two or more of them are required.

Cloud/grid computing and other form of future computing are now aiming at reducing/minimizing huge unutilized computing power of numerous computers available.

4 Impact of Emerging Engineering on RMA, CMA, UHC, and Their Teaching RMA, CMA, UHC are interconnected, domain knowledge, polynomial-time algorithms are desired

Stand-alone mathematics (RMA) is unthinkable today. Also doing RMA for academic interest only has highly limited scope. Further, each of CMA and supercomputing or, equivalently, ultra-high-speed computing (UHC) or hyper-computing cannot be stand-alone. Both are closely related so far as their real-world implementations are concerned. RMA, CMA, and UHC have to be implicitly or explicitly embedded in all sciences and engineering/technology and readily useful to the society. For this we need to have a fairly good view of the trends that are emerging in engineering during the coming decade(s). These three areas as well as their teaching need to be changed/oriented based on the foreseeable technology/ engineering activities. Consequently the domain knowledge, i.e., knowledge of the concerned technology /engineering activities, has to be acquired by the scientists involved in RMA, CMA, and UHC. All the emerging engineering activities have one thing in common. This is the demand on more and more computing power, i.e., more and more computing resources such as processing speed, band-width, and storage space (both main memory and hard disk) since these activities are becoming increasingly highly compute-, storage-, and communication-intensive. Besides, there is always a need for more efficient polynomial-time [12, 13] algorithms with optimized storage, intra-algorithm and inter-algorithm communications. Wherever such a polynomial-time algorithm is currently unavailable or such an algorithm is exponential/combinatorial [12, 13] the concerned scientists need to attempt to devise/design a polynomial-time algorithm mathematically. Also, they should devise best possible evolutionary approaches whose computational and storage complexities are polynomial-time. Here we first describe the trends in engineering / technology during the coming decade and then based on these trends we put forth how mathematics teachers should orient themselves to adopt the dynamic situation in applicable science and technology.

Emerging trends The current technology/science trends are, among others, space tourism/travel, nanotechnology, mapping of the human genomes, natural (e.g. human/living being) brain research, social media, face transplant, hybrid cars, global communications, non-silicon technology such as quantum computing technology, digital storage, sub atomic/ sub-subatomic particles as building blocks of the universe, nonmaterial (spiritual) science pervading and injecting life into matters, Cancer Cure, Synthetic biology, Bio-interfaces, Long time storage of solar, wind, and wave energy, and smart grid. Since conventional materials being used currently do not work sufficiently well, Radical materials (products based on carbon nanotubes) are becoming increasingly promising.

RMA, CMA, UHC, and their teaching should be oriented based on dynamic domain knowledge/requirements Some of the other areas, besides the aforementioned ones, which are very important for the society and are emerging in a big way are Data interface and Managing global warming. These emerging areas necessitate RMA, CMA, and UHC to orient themselves according to their dynamic needs.

It may be observed that mathematical solution is of no use to an engineer until it is translated into numbers since he will be able to implement numerical solution for building bridges/machines/mansions but not the mathematical (symbolic) solution. Any mathematics teaching should include CMA teaching. It is particularly so because UHCs are available individually with practically everybody. Mathematics teaching without CMA and UHC will neither be enjoyable nor be satisfactory to most of the students.

Lots of innovative activities as well as dynamic teaching (to produce sufficiently able human resources) in RMA, CMA and UHC are needed. Both the deterministic and evolutionary algorithms best suited for the concerned physical problems need to be designed and developed in the fast moving scenes in the years to come. The society, the world will then be highly benefitted.

RMA, CMA, and UHC ((combined together) give enormous amount of insight into the physical problem posed and hence implicitly help the physical scientists to appropriately modify the problem formulation and then re-solve in a comparatively short tolerable span of time. The new solution will expose us to the problem still further and thus recursively help us to improve the problem formulation and the successive solutions dynamically. It is necessary to stress that the current usage of a computer to solve a problem / successive modified problems is not a one-time affair as was used to be practically so a few decades ago. This is because every individual has a high-speed computer all for her unlike a single main frame (centralized) computer available to many in a time-sharing mode during 1960s, 1970s and even 1980s.

5 Conclusions

Mathematics (RMA) is viewed differently by different people over centuries. In essence, it is all-pervading and springs from the purity of mind. There are three gigantically different mathematics, viz., NMA, RMA, and CMA. While NMA is completely error-free, ever-existing, non-stoppable, and mostly beyond the comprehension of human being, RMA and CMA both have error component, are consciously stoppable, and are within the comprehension of human being.

RMA has been continuing to emulate NMA as best as possible. CMA uses UHC and the knowledge of RMA to manifest how good/close the solution/result is compared with the actual physical outcome. Twenty-first century RMA and its computational arm (CMA) along with ultra-high speed computing (UHC) have impacted immensely the on-going engineering and technological activities (ETA). The other way also, this statement is true. All of these, viz., RMA, CMA, UHC, and ETA are mutually impacting one another and are enriching themselves fast dynamically through nonstop innovations. Not only the current ETA but also the emerging ETA have affected profoundly both RMA and CMA. The exponentially galloping computing power has helped achieving all that we see in the real world today, which possibly was beyond our imagination in 1950s. Teaching RMA, CMA, and

UHC along with that of concerned domain (physics, chemistry, biology, engineering or art) is also impacted and will (will have to) continue to remain dynamic to meet the future need of trained and innovative man-power in all spheres of human activities.

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